

LV 11.4500 – UBUNG 1

Exercises from FJK. FJK 1.2.9, 1.2.12, 1.2.17, 1.3.1, 1.3.2, 1.3.4, 1.3.8,

Other exercises.

U1.1 For the discrete rv X_1, X_2, X_3 , compute the value of

$$\mathbb{P}(X_1 = a_1, X_2 = a_2, X_3 = a_3)$$

provided

$$\mathbb{P}(X_2 = a_2 \mid X_3 = a_3) = 1/3, \quad \mathbb{P}(X_1 = a_1 \mid X_2 = a_2, X_3 = a_3) = 1/2$$

and $\mathbb{P}(X_3 = a_3) = 1/7$.

U1.2 Prove that for any discrete rv $X : \mathbb{R} \rightarrow A \subset \mathbb{R}^d$ with $d > 1$ that is square-integrable, meaning $\mathbb{E}[|X|^2] < \infty$, it holds that

$$\mathbb{E}[|X - \mathbb{E}[X]|^2] \leq \mathbb{E}[|X - k|^2] \quad \forall k \in \mathbb{R}^d$$

U1.3 For discrete rv $X : \Omega \rightarrow A \subset \mathbb{R}^d$ and $Y : \Omega \rightarrow B \subset \mathbb{R}^k$, any mapping $f : \mathbb{R}^d \times \mathbb{R}^k \rightarrow \mathbb{R}$ such that $|\mathbb{E}[f(X, Y)]| < \infty$, prove that

$$\mathbb{E}[f(X, Y) \mid Y = b] = \mathbb{E}[f(X, b) \mid Y = b] \quad \forall b \in B.$$

Hint: Use that $f(X, Y) : \Omega \rightarrow C = f(A, B)$ is a discrete rv. (Alternatively, use that for any $H \in \mathcal{F}$, $\mathbb{E}[X \mid H] = \mathbb{E}[X \mathbb{1}_H] / \mathbb{P}(H)$).